Shock Waves and Drag in the Numerical Calculation of Compressible, Irrotational Transonic Flow

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Theme

BECAUSE the equations of compressible, irrotational flow admit shock waves that model Rankine-Hugoniot shocks in the transonic range, numerical procedures developed for transonic flow often employ an isentropic assumption. The shock relations of isentropic flow are found from the theory of a weak solution, and numerical experiments demonstrate that properly differenced equations do give correct jump relations. Analysis further shows that drag can be balanced against the momentum gained through an isentropic shock. This approach offers a means of computing drag that is less sensitive to numerical truncation errors than an integration of surface pressure.

Contents

Weak solutions: The equations of irrotational, inviscid, adiabatic flow define shock jump conditions which are analogous to the Rankine-Hugoniot relations. The theory of a weak solution (Lax[1]) is used to derive these relations for isentropic flow. For simplicity assume that the flow is one dimensional, then across an isentropic normal shock wave in which mass, energy, and entropy are conserved the jump condition

$$\left(1-\frac{\gamma-1}{2}u_1^{\,2}\right)^{1/(\gamma-1)}u_1=\left(1-\frac{\gamma-1}{2}u_2^{\,2}\right)^{1/(\gamma-1)}u_2 \qquad (1)$$

applies. In Eq. (1), u is the velocity nondimensional with respect to the stagnation speed of sound, and γ is the ratio of specific heats. Subscripts 1 and 2 indicate conditions before and behind the normal shock.

The discontinuous solution of Eq. (1) is shown in Fig. 1 $\{M^*=[(\gamma+1)/2]^{1/2}u\}$ and contrasted to the Prandtl relation for a Rankine-Hugoniot shock. Figure 1 clearly shows that throughout the trasonic range the isentropic jump condition is in good agreement with the Prandtl relation. However, in isentropic flow, the second law of thermodynamics cannot be used to formally exclude the expansion shock wave. Furthermore, through an isentropic shock wave momentum normal to the shock is not conserved.

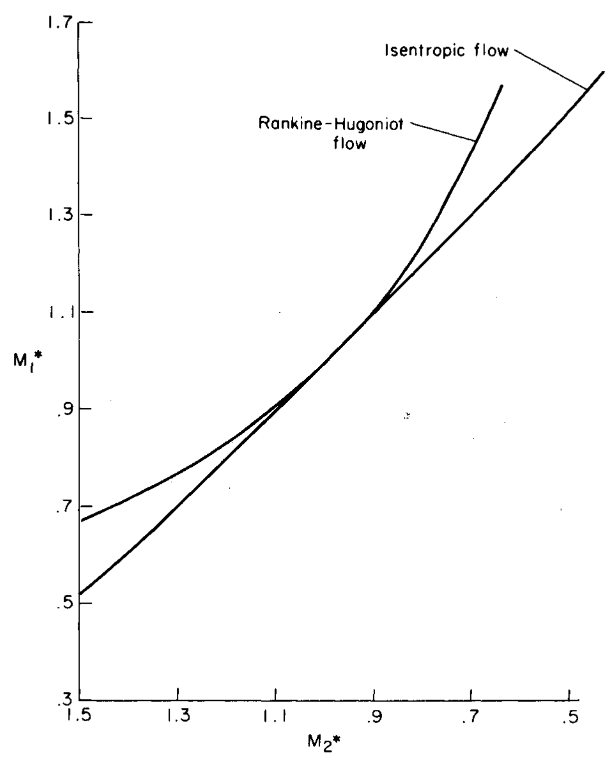

Fig. 1 Jump relations for normal shock.

Proper numerical differencing: Numerical experimentation demonstrates that if proper differencing is used current relaxation schemes give the correct jump solutions to the equations of transonic flow. Such schemes can account for multiple shocks as well as shock-free solutions if proper upwind differencing is used in supersonic regions. Furthermore, the expansion shock wave, which is a permissible mathematical solution as shown by Fig. 1, can also be obtained. If only downwind differencing is used in the supersonic region, the expansion shock is always found and the compressible shock is always excluded. Figure 2 illustrates such solutions. This and similar examples show that all correct allowable mathematical solutions can be obtained, but solutions which do not model physics can be excluded by proper choice of the differencing method.

Drag associated with an isentropic shock: The Oswatitsch drag equation[2] relates drag to the entropy production in shock waves, but in isentropic flow entropy is constant across shock waves and throughout the flow. Nevertheless, integration of the surface pressure over an airfoil with an isentropic shock does result in a drag force and this drag can be attributed to the fact that the component of momentum normal to the shock is not conserved across an isentropic shock. It can be shown from the equations of continuous isentropic flow that the integral forms expressing the conservation of mass, momentum and energy apply for arbitrary closed contours that exclude discontinuities in the flowfield. Consequently, if a suitable closed contour is taken about the airfoil and its shocks, the momentum conservation relation applies on this contour. For completely isentropic flow, conditions in the far field (including the wake) are the same as in the freestream. Hence, the contour integration of streamwise momentum about the shocks must balance the contour integral about the body. However, the integral over the body is equal to the drag, and evaluation of the integral over the isentropic shock or shocks leads to the relation

$$D=\sum_{\text{isen. shock}}\int \sin\theta\,\Delta(p+\rho q_n^{\,2})\,dA \qquad (2)$$

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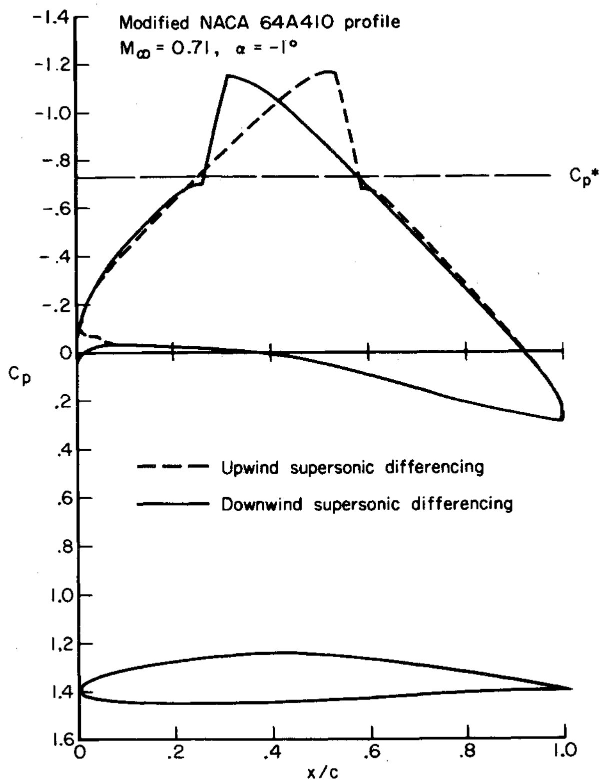

Fig. 2 Comparison of expansion and compression shocks solutions for a typical airfoil.

where D is the drag, θ the angle between the shock and the x coordinate, p and ρ the fluid pressure and density, and q_n the velocity normal to the shock surface, A. Here $\sum$ indicates that the integration is taken over every shock, and Δ denotes the difference between values of momentum before and behind the shock.

Numerical evaluation of wave drag: The fact that drag must equal the momentum gain through an isentropic shock provides another means of computing wave drag from a numerical solution that is relatively insensitive to small errors. In particular, if a numerical method using thin airfoil theory can predict the shock strength and location accurately, the wave drag can be computed correctly even if the pressure distribution is incorrect at the nose of the airfoil. To this end, a method was developed from Eq. (2) to evaluate drag using the quantities immediately upstream of the shock. The procedure is as follows for $\gamma = 1.4$: at points along the shock surface evalute r by solution of

$$r+r^2+r^3+r^4+r^5 = \frac{1-(\frac{1}{5})(u_1{}^2+v_1{}^2)}{u_1{}^2[\sin\theta-\cos\theta(v_1/u_1)]^2} \qquad (3)$$

where r requires $\gamma = 1.4$ and r is introduced because it can be readily found by standard numerical procedures since the polynomial is a monotone function in the interval from $0 < r < 1$. The contribution to the drag coefficient from each shock is then evaluated according to the relation

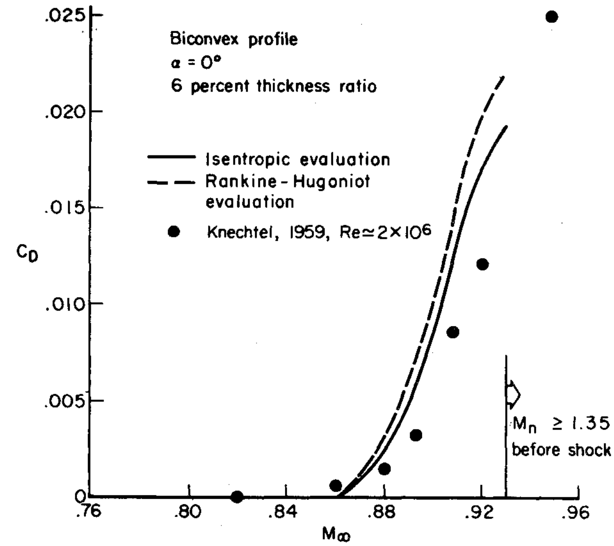

Fig. 3 Drag rise based on shock integration method.

$$C_D = \frac{2}{c}\int \Delta\mathscr{R}\left(\frac{\rho_1}{\rho_\infty}\right)\left(\frac{q_{n1}}{q_\infty}\right)^2 \sin\theta\, dz \qquad (4)$$

where

$$\Delta\mathscr{R} \equiv \frac{1}{5}(1-r^{5/2})\left[\frac{(1+r^{5/2})(1+r+r^2+r^3+r^4+r^5+r^6)}{(1+r^{7/2})r^{5/2}} - 7\right] \qquad (5)$$

and c is the chord, (ρ_1/ρ_∞) the nondimensional fluid density on the upstream surface of the shock, and (q_{n1}/q_∞) the upstream nondimensional fluid velocity normal to the shock. The distance z is the projection of the shock along the vertical coordinate, and Eqs. (3–5) are simply integrated along this upstream surface to give the drag contribution of the shock.

An example of the use of the foregoing formula is given in Fig. 3 and compared to the experimental data of Knechtel.[3] The discrepancy between the theory and the experiment is expected since the experiment is at the relatively low Reynolds number of about 2×10^6. A scheme similar to Eqs. (3–5) has also been worked out for Rankine-Hugoniot shocks and is described in the full paper. Figure 3 also contains an evaluation of C_D based on this scheme, but with the shock Mach number and location determined from the isentropic flow solution. The difference between the two curves provides an indication of the error that can be expected from use of the isentropic flow approximation.

References

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[3] Knechtel, E. D., "Experimental Investigation at Transonic Speeds of Pressure Distributions Over Wedge and Circular-Arc Airfoil Sections and Evaluation of Perforated-Wall Interference," TN D-15, 1959, NASA.